Calculation of the expansion rate of the three-volume measure in high-energy heavy-ion collisions

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In ultrarelativistic heavy-ion collisions the local three-volume measure is expanding in the longitudinal and transverse directions. This is similar to the Hubble-expansion of the universe, except that the former is not locally isotropic. As an example the expansion rate is calculated assuming that the energy-momentum tensor in the central region is that of an ideal fluid, undergoing Bjorken flow in longitudinal direction, and with initial conditions as expected for BNL-RHIC energy. While the longitudinal expansion of three-volume is independent of the energy density of the fluid, in case of 3+1 dimensional expansion the form of the hydrodynamical solution (rarefaction wave or deflagration shock) affects the three-volume expansion rate on the hadronization hypersurface. As a consequence the average expansion rate on that surface depends on the transverse size of the system. This may reflect in an impact-parameter dependence of the formation probability of light nuclei and of the freeze-out temperature of the strong interactions in the system.

In high-energy hadron-hadron or nucleus-nucleus collisions secondary particles are supposedly produced on a so-called proper-time hyperbola $\tau_0 = \sqrt{t^2 - z^2}$=const., with their number and energy density distributions independent of space-time rapidity $\eta = \frac{1}{2}\log\frac{t+z}{t-z}$ [1]. (z and t denote the longitudinal coordinate and the time measured in the global rest-frame of the reaction, respectively.) This should be a reasonable (qualitative) approximation at high energies and around midrapidity. In other words, secondary particle production occurs such that invariance under longitudinal Lorentz boosts, i.e. rapidity shifts, is obeyed. E.g., this holds true at asymptotically high energies for classical non-Abelian Yang-Mills Bremsstrahlung emitted by sources of color-charge on the light-cone [2], if recoil is neglected. The effect is also built in effective string-models for particle production at high energies: a hadron emerging from the string-decay with rapidity y_p gets on mass-shell in the global rest-frame at time $t = \tau_f \cosh y_p$, where τ_f is the formation time of the hadron in its rest-frame [3]. For a discussion within the parton model see ref. [4].

The subsequent dynamical evolution preserves the invariance of the bulk properties, e.g. the energy density distribution, under longitudinal Lorentz boosts. It is therefore convenient to switch from (t, x, y, z) to new coordinates, via [1]

$$t = \tau\cosh\eta \quad , \quad z = \tau\sinh\eta \quad , \quad x = r_\perp\cos\phi \quad , \quad y = r_\perp\sin\phi \quad . \tag{1}$$

The Minkowski line element in terms of the new coordinates is $\mathrm{d}s^2 = \mathrm{d}\tau^2 - \tau^2\mathrm{d}\eta^2 - \mathrm{d}r_\perp^2 - r_\perp^2\mathrm{d}\phi^2$, i.e. $g_{\mu\nu} = \mathrm{diag}(1, -\tau^2, -1, -r_\perp^2)$. In the following, vectors will be written in the

basis were the components 0, 1, 2, 3 correspond to the τ, η, $r_\perp$, ϕ direction, respectively. Furthermore, rotational symmetry around the longitudinal direction is assumed.

Given some boundary conditions on the hypersurface $\tau=\tau_0$, the evolution within the forward light-cone is governed by [5]

$$\partial\cdot T=0 \quad , \quad \partial\cdot N_B=0 \quad . \tag{2}$$

In the ideal fluid approximation (natural units are employed, $\hbar=c=k_B=1$)

$$T^{\mu\nu}=(\epsilon+p)u^\mu u^\nu-pg^{\mu\nu} \quad , \quad N_B^\mu=n_B u^\mu \quad . \tag{3}$$

The four-velocity of the locally comoving frame is normalized to $u\cdot u=1$, and is given by $u^\mu=\gamma_\perp(1,0,v_\perp,0)$, where $\gamma_\perp^{-2}=1-v_\perp^2$ [6–9]. The projections of the equations for the energy-momentum tensor parallel and orthogonal to u, and the continuity equation for the net baryon current yield

$$u\cdot\partial\,\epsilon=-\left(\epsilon+p\right)\partial\cdot u \quad , \tag{4}$$

$$\partial_\eta p=0 \quad , \tag{5}$$

$$u\cdot\partial\,n_B=-n_B\,\partial\cdot u \quad , \tag{6}$$

with

$$u\cdot\partial=\gamma_\perp\left(\partial_\tau+v_\perp\partial_\perp\right) \quad . \tag{7}$$

The expansion scalar $\partial\cdot u$ is given by

$$\partial\cdot u\equiv\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}u^\mu\right)=\frac{\partial\gamma_\perp}{\partial\tau}+\frac{\gamma_\perp}{\tau}+\frac{u_\perp}{r_\perp}+\frac{\partial u_\perp}{\partial r_\perp} \quad . \tag{8}$$

Thus, for purely longitudinal expansion ($u_\perp=0$, $\gamma_\perp=1$), we simply have $\partial\cdot u=1/\tau$. This is obvious because in this case $V=\pi R_\perp^2\tau$ is the three-volume on a $\tau=$ const. hypersurface corresponding to a length in space-time rapidity of $\Delta\eta=1$. In other words, the Hubble constant in longitudinal direction is

$$H(\tau)=\frac{1}{\tau} \quad , \tag{9}$$

for $\tau\geq\tau_0$ and $r_\perp\leq R_\perp$. $H(\tau_0\sim 0.5-1\text{ fm})$ is roughly 10^{40} times bigger than the present rate of expansion of the universe, and about 10^{18} times the Hubble constant at the cosmological hadronization phase transition. Note that for purely longitudinal expansion $\partial\cdot u$ (or H) is independent of ϵ and n_B, i.e. the fluid does not influence the expansion rate. This is due to eq. (5): free (non-accelerated) flow always means $\eta=$ const., *independently* of the equation of state $p(\epsilon,n_B)$.

As a side-remark, note that for *spherically* symmetric, three-dimensional boost-invariant expansion, $\vec{v}=\vec{r}/t$. One finds $\partial\cdot u=3/\tau$, where (1) is now replaced by $t=\tau\cosh\eta$, $r=\tau\sinh\eta$, that is $\tau^2=t^2-r^2$ and $\eta=\frac{1}{2}\log\frac{t+r}{t-r}$. Again, the expansion scalar $\partial\cdot u$ is independent of the equation of state.

One also observes from eqs. (4,6) that the continuity equations (2) can not be extrapolated to $\tau = 0$ because the three-volume vanishes (the Jacobian $|(\partial(t,z)/\partial(\tau,\eta)| = \tau$ of (1) is zero). Nevertheless, the transformation from (t,z) to (τ,η) is not useless, because the classical description breaks down for $\tau \to 0$ anyway. In such reactions the $\langle p_\perp \rangle$ of produced quarks and gluons are on the order of 1 GeV, and thus the uncertainty relation sets a time-scale of $\sim 1/\langle p_\perp \rangle = 0.2$ fm, below which this approach must fail.

If transverse expansion is superimposed on longitudinally boost-invariant expansion, $\partial \cdot u$ of course depends on the transverse flow velocity, cf. eq. (8), which by itself depends on transverse pressure gradients (caused by energy density and/or baryon density gradients). Thus, the evolution of ϵ, n_B, u, and $\partial \cdot u$ is coupled via eqs. (4,6). Consider, in particular, a hypersurface $\sigma^\mu = (\tau, \eta, r_\perp, \phi)$ in space-time (e.g. a surface of constant time, or temperature, or the hadronization hypersurface $\lambda = 0$, where λ denotes the local fraction of quark-gluon phase). In parametric representation, σ^μ is a function of three parameters [5]. In our case, due to the symmetry under rotations around and Lorentz-boosts along the beam axis, two of these parameters can simply be identified with η and ϕ, while τ and $r_\perp$ depend only on the third parameter, call it ζ. Thus, $\zeta \in [0,1]$ parametrizes the hypersurface in the planes of fixed η and ϕ (counter clock-wise). Then, the normal is[1]

$$\mathrm{d}\sigma_\mu = \epsilon_{\mu\alpha\beta\gamma} \frac{\partial \sigma^\alpha}{\partial \zeta} \frac{\partial \sigma^\beta}{\partial \eta} \frac{\partial \sigma^\gamma}{\partial \phi} \mathrm{d}\zeta \mathrm{d}\eta \mathrm{d}\phi = r_\perp \tau \left(-\frac{\partial r_\perp}{\partial \zeta}, 0, \frac{\partial \tau}{\partial \zeta}, 0 \right) \mathrm{d}\zeta \mathrm{d}\eta \mathrm{d}\phi \tag{10}$$

The three-volume measure on the hypersurface is

$$\mathrm{d}V \equiv \mathrm{d}\sigma \cdot u = r_\perp \tau \left(u_\perp \frac{\partial \tau}{\partial \zeta} - \gamma_\perp \frac{\partial r_\perp}{\partial \zeta} \right) \mathrm{d}\zeta \mathrm{d}\eta \mathrm{d}\phi \quad . \tag{11}$$

E.g., on constant-τ hypersurfaces and for purely longitudinal flow, $\mathrm{d}V = \tau \mathrm{d}^2 r_\perp \mathrm{d}\eta$. Note that $\partial \cdot u$ is simply the rate of expansion of the three-volume measure (11), since

$$u \cdot \partial \, \mathrm{d}V = \mathrm{d}V \, \partial \cdot u \quad . \tag{12}$$

Eq. (12) can be verified by an explicit calculation using the relations (7,8,11). However, the following proof is simpler and more general (it does not assume longitudinal boost-invariance and cylindrical symmetry). Note that the total net baryon number

$$\mathcal{B} \equiv \int \mathrm{d}\sigma_\mu N_B^\mu = \int \mathrm{d}V n_B \tag{13}$$

is a constant, and thus $u \cdot \partial \mathcal{B} = 0$. Therefore,

$$\int \left(u \cdot \partial \mathrm{d}V \right) n_B = u \cdot \partial \int \mathrm{d}V n_B - \int \mathrm{d}V u \cdot \partial n_B = \int \mathrm{d}V n_B \left(\partial \cdot u \right) \quad . \tag{14}$$

[1] In our basis $\epsilon_{\mu\alpha\beta\gamma}$ contains a factor $\sqrt{-g} = r_\perp \tau$.

In the second step, the continuity equation (6) has been used. This must be true for any arbitrary function n_B, and therefore the identity (12) must hold.

To discuss a specific example, equations (2) have been solved numerically employing the finite-difference scheme RHLLE, as described and tested in [10]. The initial conditions were chosen as might be appropriate for collisions of heavy nuclei at BNL-RHIC energy, $\sqrt{s} = 200$ GeV per incident nucleon pair. The main goal [11] of these experiments is to repeat, in the laboratory, the QCD phase transition that occured at some stage in the universe. As already mentioned above, the difference is that the "Hubble-constant" is much larger in high-energy heavy-ion collisions. The energy density is expected [11,12] to be significantly higher than in lower-energy reactions, such that the expansion effect, and the influence of the QCD hadronization phase transition, should be more prominent.

In particular, on the $\tau = \tau_0$ hypersurface, an entropy per net baryon of $s/n_B = 200$ is assumed; see [9] for the resulting single-particle transverse momentum spectra as well as average transverse momenta and velocities of various hadrons. For simplicity, s and n_B are assumed to be homogeneously distributed on the $\tau = \tau_0$ hypersurface, independent of η and ϕ, and proportional to a step function, $\Theta\left(R_\perp - r_\perp\right)$. Except close to the light-cone, where the flow velocities are largest but where the fluid is very dilute, the bulk dynamics is not very sensitive to the precise initial profile [9].

Estimates for τ_0 at RHIC energy span the range $0.2 - 1$ fm [12]. Here, the average value of $\tau_0 = 0.6$ fm is employed. For the given initial conditions, and for the MIT bag-model equation of state (EoS) described below, the initial temperature is $T(\tau_0) \approx 300$ MeV.

To close the system of equations (2) one has to specify an EoS, which determines the function $p(\epsilon, n_B)$. In the low-temperature region a gas of non-interacting hadrons that includes all known [13] strange[2] and non-strange hadrons and hadronic resonances up to a mass of 2 GeV is assumed. At high temperatures, the EoS is that of an ideal gas of quarks, antiquarks (with masses $m_u = m_d = 0$, $m_s = 150$ MeV), and gluons. In addition, a bag term [14] $+B$ in the energy density, and $-B$ in the pressure is included. This is the same as adding a term $Bg^{\mu\nu}$ to the energy-momentum tensor of the quark-gluon fluid. Mathematically, the bag term resembles the cosmological-constant term which is introduced in some cosmological models [5]. However, since it is added to the energy-momentum tensor of the quark-gluon fluid only, and not to that of the hadronic fluid, it changes the form of the hydrodynamical solution (deflagrations can occur, see below).

Finally, the phase coexistance region is obtained from Gibbs' conditions of phase equilibrium. Thus, the EoS exhibits a first-order phase transition. The value of B is determined by T_C; for $T_C = 160$ MeV, which is roughly in accord with recent lattice-QCD results [15], one finds $B = 380$ MeV/fm^3.

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[2]The strangeness chemical potential is determined by the requirement that the net strangeness density n_S, and thus the strangeness current $N_S^\mu \equiv n_S u^\mu$, vanish everywhere.

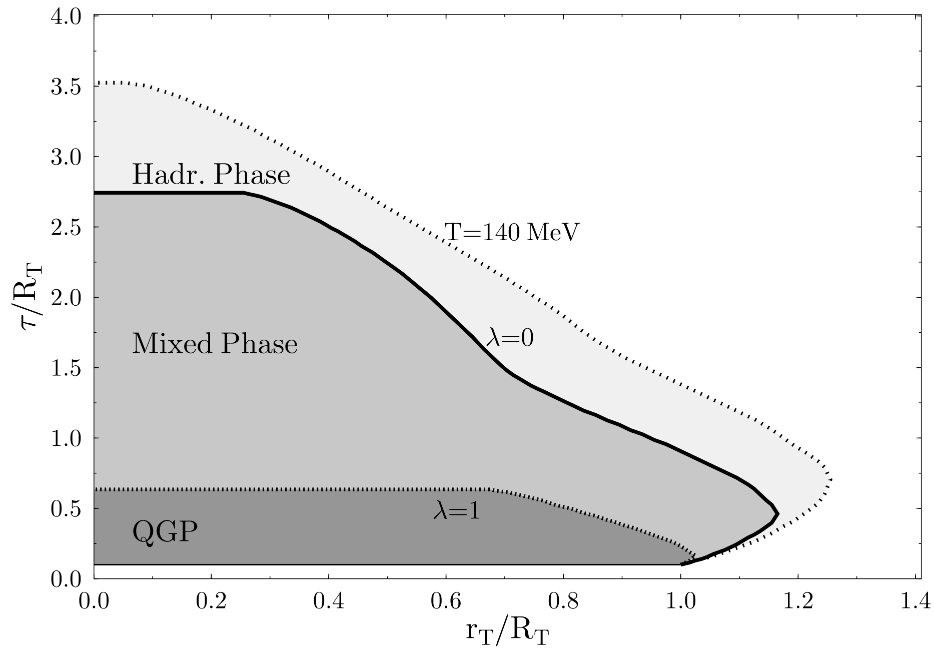

FIG. 1. Hypersurfaces corresponding to $\lambda = 1$ (boundary between QGP and mixed phase), $\lambda = 0$ (boundary between mixed phase and pure hadron phase, i.e. the hadronization hypersurface), and the $T = 140$ MeV isotherm; for $\tau_0/R_\perp = 1/10$.

Fig. 1 shows the hypersurfaces where the pure QGP and the mixed phase end, respectively, as well as the $T = 140$ MeV isotherm. One observes a rarefaction wave developping at $r_\perp = R_\perp$, $\tau = \tau_0$. It accelerates a part of the quark-gluon fluid in transverse direction, before the boundary between quark-gluon plasma and mixed phase is reached ($\lambda = 1$). Those parts of the system which have not been affected by transverse expansion enter the mixed phase at the same time, which is the horizontal part of the $\lambda = 1$ hypersurface at $\tau/R_\perp \simeq 0.6$. The expansion rate within those space-time regions is simply $1/\tau$, as discussed above.

During the transition to hadronic matter, entropy is converted from the quark-gluon to the hadronic phase at a rate which of course depends on $\partial \cdot u$. For example, assume for simplicity a net baryon free fluid, and that shock-solutions do not occur, such that entropy is conserved. Then, the continuity equation for the entropy four-flow, $\partial \cdot (su) = 0$, yields

$$u \cdot \partial \, \lambda = -\left(\partial \cdot u\right) \left(\lambda + \frac{s^H}{s^Q - s^H}\right) \quad . \tag{15}$$

s^H, s^Q denote the entropy densities of the hadronic and quark-gluon fluids at $T = T_C$. λ is the local fraction of quarks and gluons within the mixed phase, such that the total entropy density is $s = \lambda s^Q + (1 - \lambda) s^H$. Eq. (15) shows that the rate of adiabatic conversion of quark-gluon fluid into hadronic fluid is governed by the three-volume expansion rate $\partial \cdot u$, which is larger than $1/\tau$ in the space-time regions where transverse expansion is active. Therefore, hadronization is faster close to the surface, $r_\perp \sim R_\perp$, than in the interior, cf. Fig. 1. In particular, the local expansion rate of the hadronization volume ($\lambda = 0$

hypersurface) determines whether the emerging hadrons can maintain local equilibrium or not.

Fig. 2 shows the expansion factor $\xi \equiv \tau\partial \cdot u$ as a function of τ and $r_\perp$. $\partial \cdot u$ is multiplied by τ to have $\xi = 1$ in space-time regions where the expansion is purely longitudinal. In the front-left corner one can see the expansion induced by the rarefaction in the quark-gluon fluid. Once the mixed phase is reached, the rarefaction wave "stalls".

This is due to the very small velocity of sound in the phase coexistance region. For recent discussions of the consequences of this effect in cosmology see e.g. [16]. In heavy-ion collisions, it leads to "stall" of the transverse expansion [7–9]. Fig. 2 shows that therefore ξ is nearly time independent until hadronization is completed and the purely hadronic phase is reached ($\lambda = 0$): the curves of constant expansion rate (right panel of Fig. 2) at $r_T \approx 0.6$ and 0.7 are nearly parallel to the time-axis.

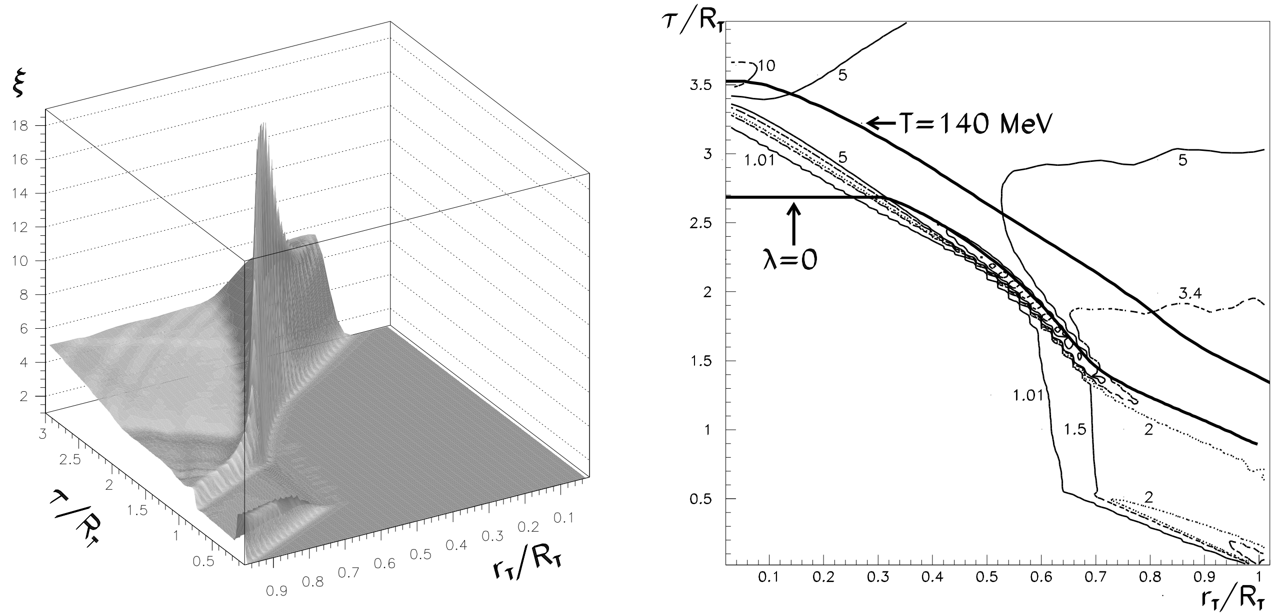

FIG. 2. Left: the expansion factor $\xi \equiv \tau\partial \cdot u$ as a function of τ and $r_\perp$. Right: curves of constant ξ, labeled by the value of ξ to which they correspond to; the thick lines depict the hadronization hypersurface, $\lambda = 0$, and the $T = 140$ MeV isotherm, respectively. Both for $\tau_0/R_\perp = 1/10$.

The hadronization hypersurface ($\lambda = -0$, i.e. infinitesimally smaller than 0) and the $T = 140$ MeV isotherm from Fig. 1 are shown again in Fig. 2. Clearly, the expansion rate on these hypersurfaces is *not constant*, i.e., they are not hypersurfaces of homogeneity. This is in contrast to an isotropically expanding homogeneous universe [17]. Thus, in very high-energy heavy-ion collisions it does not seem natural to assume decoupling on a hypersurface of constant temperature, as already pointed out in [18].

For the equation of state with a first-order phase transition, hadronization is accomplished via a deflagration shock [7,8,10,19]. (For the geometry at hand, the shock front is actually curved, as can be seen in Fig. 2). The final-state of the shock is the steady-state

corresponding to the Chapman-Jouget point [10,20] (maximum entropy production). The hadronization point $\lambda = 0$, however, is in general located on the shock (and has higher pressure than at the CJ-point). On the shock front $\xi \to \infty$ for an ideal fluid. In reality, a finite viscosity will smear out the shock front slightly; in the figure, ξ remains finite because in the numerical solution the derivatives in eq. (8) are replaced by finite differences. Behind the shock front the expansion proceeds again via simple rarefaction waves, and ξ is finite.

The large expansion rate of the hadronization three-volume could open the interesting possibility that in the vicinity of that hypersurface the fluid breaks up into smaller droplets [21] which decouple from each other. This can lead to rapidity fluctuations [21,22] or even the formation of droplets of disoriented chiral condensate [23]. However, this scenario will not be discussed here in detail.

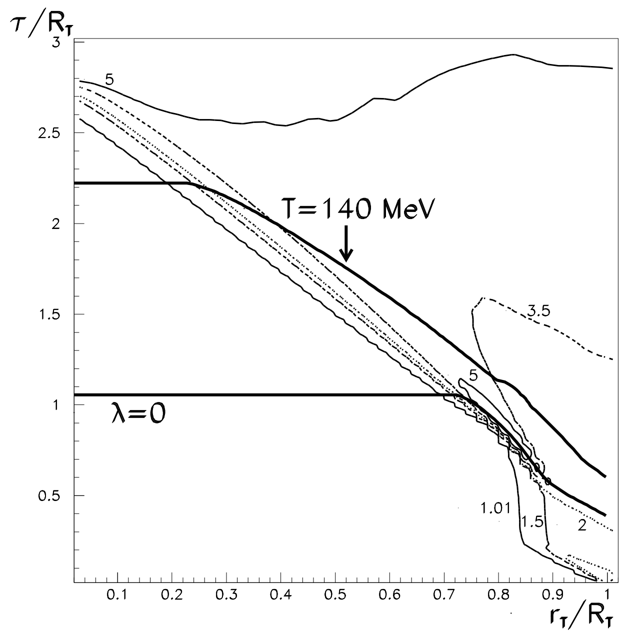

FIG. 3. Curves of constant ξ (for $\tau_0/R_\perp = 1/25$, i.e. larger system than in Fig. 2) labeled by the value of ξ to which they correspond to; the thick lines depict the hadronization hypersurface, $\lambda = 0$, and the $T = 140$ MeV isotherm, respectively.

Clearly, those three-volume elements of the hadronization hypersurface that lie on the shock front will expand rather rapidly, even if the shock-width were slightly smeared out by a finite viscosity. It is interesting to note that for the smaller initial radius one obtains $\xi > 1$ almost over the entire hadronization hypersurface (and behind it), while for large $R_\perp$, cf. Fig. 3, $\xi = 1$ over a large part of the hadronization hypersurface. For very large systems the expansion scalar $\partial \cdot u$ approaches the "trivial", EoS independent form $\partial \cdot u = 1/\tau$.

Of course, the question arises how the large expansion rate affects the observed spectra of hadrons. The most pronounced effect would be expected in space-time regions where the expansion scalar is large and the fluid is already rather dilute. E.g., the decoupling from local thermal equilibrium should be influenced [18,24,25]: a large $\partial \cdot u$ hinders scattering between

the particles of the fluid. Therefore, the produced hadrons should decouple at higher T as compared to the case of small expansion rate, cf. also the discussion in [18,24]. In other words, the strong interaction freeze-out hypersurface should approach the hadronization hypersurface as beam energy increases (as observed in [26]) and as the initial radius decreases.

If indeed the strong interactions cease at higher temperature as compared to a slowly expanding three-volume measure, the average phase-space density of frozen-out pions or kaons should also increase. This quantity can be estimated from measurements of the two-particle correlation function [27]. Moreover, the synthesis of light nuclei (and anti-nuclei) might be suppressed if the three-volume measure expands rapidly, because it becomes more difficult to coalesce (anti-)nucleons into clusters. Also, unlike in a static fireball in global thermodynamical equilibrium one may expect that the decoupling conditions of various hadron species differ: a large volume expansion rate should lead to earlier decoupling of hadron species with small cross section [26]. Finally, if the QCD chiral phase transition is of first-order only at larger baryon-chemical potential μ_B [28], but a smooth cross-over at small μ_B, the hydrodynamical solution (and thus the expansion rate) might look very different on the two sides of the critical point. In the region where the phase transition is first order, a shock can develop and lead to a rather large expansion rate. In the cross-over region, on the other hand, only rarefaction waves but no shocks will form, and the average expansion rate at chiral symmetry restoration should be smaller. As discussed above, this difference might reflect in the freeze-out properties of the strongly interacting system.

To analyze these points in detail, one has to supplement the fluid-dynamical solution on the hadronization hypersurface with a more detailed kinetic treatment of the hadronic stage, which explicitly accounts for the various (elastic and inelastic) elementary hadron-hadron scattering processes [26]. One might then be able to study the effect of the three-volume expansion on the evolution and freeze-out properties of the hadron fluid. Those calculations found that some hadrons are indeed emitted directly from the hadronization hypersurface without scattering any further, and that the large expansion rate almost "freezes" the chemical composition of the hadron fluid. More detailed studies, e.g. with varying $\tau_0/R_\perp$, will be performed in the future.

Similar studies could be done in the laboratory by varying the initial transverse size. One should keep in mind, however, that the energy density in the central region decreases with mass number, which could partly counterbalance the effect.

In summary, transverse expansion couples the scalar $\partial \cdot u$, where u is the four-velocity of the locally comoving frame, and which can be interpreted as the expansion rate of the local three-volume measure, to the properties of the fluid (energy and baryon density and the EoS). If hadronization proceeds via a shock wave, the expansion rate can become particularly large on the hadronization hypersurface. The hadrons produced on such a shock may decouple immediately. The average expansion rate on the hadronization hypersurface depends on the size of the system (at fixed initial energy density). This may reflect in a system-size dependence of the coalescence probability of light (anti-)nuclei and of the freeze-out properties of the strongly interacting system (average phase space density and temperature).

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